BMATH MIDTERM EXAMINATION ALGEBRAIC GEOMETRY

Attempt all questions. Assume that the base field k, in all questions below, is algebraically closed. Total: 50 marks. Time: 3 hours

(1) Consider the set $X = \{(1,0),(-1,0)\} \subset \mathbb{A}^2_k$. Calculate $I(X)$. (7 marks)
(2) Let $X = Z(xy-1)$ and $Y = Z(y-x^2)$ both considered as algebraic subsets of $\mathbb{A}^2_k$. Is X isomorphic to Y? Justify your answer. (7 marks)
(3) Describe the ring of regular functions on the open set $D(x-4y^2) \subset \mathbb{A}^2_k$. (8 marks)
(4) Define a regular function on an open subset of a projective variety and prove that the only regular functions on $\mathbb{P}^n_k$ are constants. (8 marks)
(5) State the Weak Nullstellensatz and Hilbert's Nullstellensatz. Prove that the former implies the latter. (2+2+6 = 10 marks)
(6) Define a morphism between two affine algebraic sets $X \subset \mathbb{A}^n_k$ and $Y \subset \mathbb{A}^m_k$. Prove that there is a natural bijection between the set of all morphisms $X \to Y$ and all k algebra homomorphisms $A(Y) \to A(X)$. (2+8 = 10 marks)